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Spin wave excitations of a magnetic pillar with dipolar coupling between the layers

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14. ABSTRACT

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1. Introduction

The physics of magnetic multi-layered systems is very rich, and these systems have a lot of practical applications. It is well known that the dipole-dipole interaction in such systems can play a crucial role for the determination of both the ground state (e.g., parallel/antiparallel configuration) and the spectra of elementary magnetic excitations – spin wave modes. *Continuous* multi-layered ferromagnetic thin films have been extensively studied for more than thirty years. Recent developments in nano-lithography allowed one to investigate magnetization dynamics in *nano-patterned* magnetic structures – thin single- and multi-layered magnetic elements with lateral sizes below 1 μm. The examples of such structures are ordered arrays of long magnetic stripes with micro-metric width [1, 2] and arrays of long magnetic stripes situated above a continuous magnetic film [3].

Another class of nano-patterned magnetic systems where dipole-dipole interaction plays an important role are nano-scale spin-torque oscillators (STO). Since the first observations of microwave generation by STO [4], extensive experimental work have been reported concerning the frequency measurements of this generation in low-amplitude regime for the large variety of magnetic multi-layer nano-structures. Typically, the experimentally observed dependence of the measured frequency on the bias magnetic field can be well described by the traditional Kittel' expression for both in-plane and perpendicularly magnetized cases, provided that the saturation magnetization is decreased by 30 to 75 percent depending on a system [4, 5, 6, 7, 8]. This apparent reduction of the saturation magnetization in nano-patterned systems was sometimes attributed to the effect of patterning and sometimes to the influence of the dipole-dipole interaction between the magnetic layers. To the best of our knowledge, however, such influence has never been systematically studied theoretically, and it is unclear whether the dipole-dipole interaction alone can lead to a such substantial apparent reduction of the saturation magnetization.

Micro- and nano-structured ferromagnets can also be efficiently used as microwave absorbers in monolithic microwave integrated circuit due to the fact that their ferromagnetic resonance (FMR) frequency can be tuned in the range of several tens of gigahertz by applying external magnetic fields. However, the application of large external magnetic fields is in many cases inconvenient, and it has been recently proposed to control the FMR frequency using the dipolar magnetic field in patterned magnetic multi-layers [9, 10]. This approach, again, requires a deeper understanding of the influence of the dipole-dipole interaction on the spectrum of spin wave excitations in magnetic layered and patterned structures.

In the present paper we theoretically studied the influence of the dipole-dipole interaction on the spatially-uniform spin wave modes in an in-plane magnetized two-layered magnetic nano-pillar shown on figure 1. We developed a simple analytical formalism of accounting the mutual dipolar interaction between the magnetic nano-elements which can be used to study spin wave excitations in arbitrary patterned magnetic system. We calculated frequencies of coupled spin wave modes in a two-layered magnetic nano-pillar and demonstrated that these frequencies can be described using Kittel-like expressions with renormalized saturation magnetization and bias magnetic field. We demonstrated that the dipolar interaction between the layers, indeed, leads to an apparent reduction of the saturation magnetization, but this effect in many cases is not sufficiently large to explain completely the observed reduction of saturation magnetization in typical experiments with nano-pillar STOs.

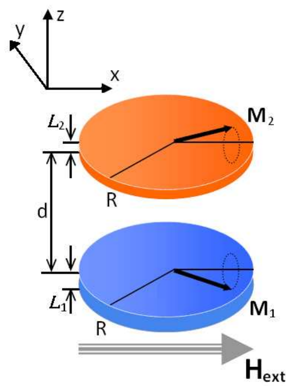

Figure 1. (Color online) Nano-pillar with two ferromagnetic layers coupled by the dipole-dipole interaction in constant bias magnetic field $\mathbf{H}_{ext}$, directed along the x-axes. Thicknesses of the layers $L_{1,2}$ and the distance d between them are much smaller than the layers radii R. The layers have the same saturation magnetization $M_1 = M_2 = M$ ($4\pi M = 8$ kOe, which is typical for permalloy).

2. Formulation of the problem and general formalism

We consider magnetization dynamics of a magnetic nano-pillar schematically shown in figure 1. The pillar consists of two thin (thicknesses L_1 and L_2) circular magnetic disks of the same radius R, separated by a non-magnetic spacer of the thickness d. We assume that the distribution of magnetization in each layer is spatially-uniform, which is justified for relatively small lateral sizes of the nano-disks ($R \leq 50$ nm). Although our approach is valid for any geometrical shape of the pillar, in actual calculations we will assume that the thicknesses of the disks $L_{1,2}$ and the non-magnetic spacer d are much smaller than the disks' radii R. For simplicity, we neglect crystallographic anisotropy of the magnetic layers and assume that the saturation magnetizations of both layers are equal, $M_1 = M_2 = M$. Also, below we will consider only the case of an in-plane magnetized magnetic nano-pillar with the external magnetic field $\mathbf{H}_{ext}$ applied along the x-axis (see figure 1).

To find the frequencies of coupled spin wave excitations in the above described magnetic pillar, it is sufficient to consider only the conservative dynamics of magnetization (i.e., to neglect terms that describe energy dissipation and excitation of spin wave modes). In this case the dynamics of two-layered magnetic nano-pillar is described by a system of two conservative Landau-Lifshits equations for the magnetization vectors $\mathbf{M}_j$ ($j = 1, 2$) of magnetic layers:

$$\frac{\partial \mathbf{M}_j}{\partial t} = \gamma \left[\mathbf{H}_{eff,j} \times \mathbf{M}_j \right] \, , \tag{1}$$

where $\gamma \approx 2\pi \cdot 2.8$ MHz/Oe is the modulus of the gyromagnetic ratio and the effective magnetic fields $\mathbf{H}_{eff,j}$ can be written as

$$\mathbf{H}_{eff,j} = \mathbf{H}_{ext} + \sum_{k=1}^{2} \mathbf{H}_{j,k} \ . \tag{2}$$

Here $\mathbf{H}_{ext}$ is the external bias magnetic field and $\mathbf{H}_{j,k}$ is the magnetodipolar field, which is created by the k-th magnetic layer and acts on the j-th magnetic layer. Since we are considering only the spatially-uniform excitations, the field $\mathbf{H}_{j,k}$ should be understood as a real spatially-nonuniform field $\mathbf{H}_k(\mathbf{r})$, created by the k-th layer and *averaged* over the volume V_j of the j-th layer:

$$\mathbf{H}_{j,k} = \frac{1}{V_j} \int\limits_{V_j} \mathbf{H}_k(\mathbf{r}_j) d^3\mathbf{r}_j \ . \tag{3}$$

The magnetodipolar field $\mathbf{H}_k(\boldsymbol{r})$, created by the *spatially-uniform* magnetization distribution $\mathbf{M}_k$ of the k-th layer, can be written in the form

$$\mathbf{H}_k(\mathbf{r}) = -4\pi \widehat{N}_k(\mathbf{r}) \mathbf{M}_k \,, \tag{4}$$

where the position-dependent demagnetization tensor $\widehat{N}_k(\mathbf{r})$ is given by [11, 12]:

$$\left(\widehat{N}_k(\mathbf{r})\right)_{sp} = -\frac{1}{4\pi} \cdot \frac{\partial^2}{\partial x_s \partial x_p} \int_{V_k} \frac{d^3\mathbf{r}_k}{|\mathbf{r} - \mathbf{r}_k|} \ . \tag{5}$$

This expression for the magnetodipolar field $\mathbf{H}_k(\mathbf{r})$ is valid both inside and outside of the k-th layer.

Using equations (3)–(5), one can write the effective magnetic field (2) in j-th layer in the concise form

$$\mathbf{H}_{eff,j} = \mathbf{H}_{ext} - 4\pi \sum_k \widehat{N}_{jk} \mathbf{M}_k \,, \tag{6}$$

where the tensors $\widehat{N}_{jk}$ are given by

$$\widehat{N}_{jk} = \frac{1}{V_j} \int\limits_{V_j} \widehat{N}_k(\mathbf{r}_j) d^3\mathbf{r}_j \ . \tag{7}$$

It is clear, that for $k = j$ the tensor $\widehat{N}_{jj}$ (*self*-demagnetization tensor) coincides with the standard effective demagnetization tensor for the j-th magnetic layer. The tensors $\widehat{N}_{jk}$ for $j \neq k$ (*cross*-demagnetization tensors) describe the mutual dipolar coupling between the j-th and k-th layers.

It should be noted that one can use equation (6) to describe the dipolar coupling between the magnetic elements of *any* shape, as long as the spin wave excitations are considered to be spatially uniform within each element. Independently of the shape of the magnetic elements, the self- and cross-demagnetization tensors $\widehat{N}_{jk}$ are symmetric, and their traces are given by

$$\mathtt{Tr}\left(\widehat{N}_{jk}\right) = \delta_{jk} \,,$$

where δ_{jk} is the Kronecker symbol, and different cross-demagnetization tensors are related by the expression:

$$V_j \widehat{N}_{jk} = V_k \widehat{N}_{kj} \ .$$

These properties directly follow from the definition (7) and, from the physical point of view, reflect the conservative nature of the magnetodipolar interaction.

Due to the azimuthal symmetry of the considered system of two disks (see figure 1), demagnetization tensors $\widehat{N}_{jk}$ are diagonal and can be represented as follows:

$$\widehat{N}_{jk} = \begin{pmatrix} \rho_{jk} & 0 & 0 \\ 0 & \rho_{jk} & 0 \\ 0 & 0 & \delta_{jk} - 2\rho_{jk} \end{pmatrix} , \tag{8}$$

i.e. each tensor depends only on one parameter ρ_{jk}. Using the definition (7), the dipolar parameters ρ_{jk} can be expressed in terms of six-fold multiple integrals as follows:

$$\rho_{jk} = \frac{1}{8\pi V_j} \int_{V_j} d^3\mathbf{r}_j \int_{V_k} d^3\mathbf{r}_k \left(\frac{3(z_j - z_k)^2 - |\mathbf{r}_j - \mathbf{r}_k|^2}{|\mathbf{r}_j - \mathbf{r}_k|^5} \right) .$$

This integral can be taken over the area of the first and the second layers after the introduction of new integration variables, one of which is the difference between the in-plane radius-vectors of the first and the second layers and the other one is the in-plane radius-vector in one of the layers. Then, the integral for the parameter ρ_{jk} can be simplified to

$$\rho_{jk} = \frac{1}{2\pi} \cdot \frac{L_k}{R} \int_0^1 dz_j \int_0^1 dz_k f\left(\frac{L_j}{R} z_j + \frac{L_k}{R} z_k + \frac{d}{R} \right) \tag{9}$$

for $j \neq k$ and to

$$\rho_{jj} = \frac{1}{2\pi} \cdot \frac{L_j}{R} \int_0^1 dz (1 - z) f\left(\frac{L_j}{R} z \right) \tag{10}$$

for $j = k$. Here

$$f(\alpha) = \left[\frac{2 + \alpha^2}{\alpha} K\left(-\frac{4}{\alpha^2} \right) - \alpha E\left(-\frac{4}{\alpha^2} \right) \right] , \tag{11}$$

where $K(\xi)$ and $E(\xi)$ are the complete elliptic integrals of the first and second kind, respectively.

The function $f(\alpha)$ has a logarithmic singularity when $\alpha \to 0$. Extracting this singularity and retaining the terms up to $O(\alpha^2 \ln(\alpha))$ one obtains an approximate analytic expressions for the dipolar parameters ρ_{jk} in the case of thin disks ($L_{1,2} \ll R$ and $d \ll R$):

$$\begin{aligned} \rho_{jk} = \frac{C}{2\pi} \frac{L_k}{R} - \frac{1}{4\pi (L_j/R)} \Bigg[& F\left(\frac{d}{R} \right) - F\left(\frac{d + L_j}{R} \right) \\ & - F\left(\frac{d + L_k}{R} \right) + F\left(\frac{d + L_j + L_k}{R} \right) \Bigg] \end{aligned} \tag{12}$$

for $j \neq k$ and

$$\rho_{jj} = \frac{1}{2\pi} \frac{L_j}{R} \left[C - \ln\left(\frac{L_j}{R} \right) \right] \tag{13}$$

for $j = k$. Here $C = -1/2 + \ln(8) \approx 1.58$ and $F(\zeta) = \zeta^2 \ln|\zeta|$. It should be noted, that (13) can be obtained from (12) by assuming that $L_k = L_j$ and $d = -L_j$.

Dimensionless parameters ρ_{jk} provide a convenient measure for the magnetodipolar coupling between the layers (or, in a general case, between arbitrarily shaped magnetic elements). In the limit of very thin disks $\rho_{jk} \to 0$, and the dipolar coupling between the layers vanishes.

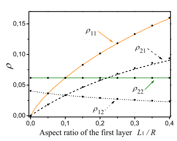

Figure 2. (Color online) The dimensionless dipolar parameters ρ_{jk}, which define the self- and cross-demagnetization tensors for the system of two thin disks coupled by dipole-dipole interaction, as functions of the aspect ratio of the first disk L_1/R. The aspect ratio of the second disk and the distance between the disks are fixed: $L_2/R = 1/10$, $d/R = 1/25$. Solid, dashed, and dotted lines were calculated from the approximate analytical expressions (12) and (13). Dots were obtained from the exact expressions (9) and (10).

Figure 2 shows how the parameters ρ_{jk} depend on the aspect ratio of the first layer L_1/R. The aspect ratio of the second layer and the distance between the layers were kept fixed, $L_2/R = 1/10$ and $d/R = 1/25$. One can see that for the typical parameters of a nano-pillar $\rho_{jk} \sim 0.05$, which corresponds to the amplitude of the dipolar magnetic field equal to $4\pi\rho_{jk}M \sim 400$ Oe. This field is larger than the ferromagnetic resonance linewidth in typical ferromagnetic materials (one can also compare the dimensionless coupling parameter $\rho_{jk} \sim 0.05$ with the dimensionless Gilbert damping parameter $\alpha_G \sim 0.01$). This means, that the damping processes can not destroy the dipolar coupling between the magnetic layers, and the spin wave modes excited in the structure shown in figure 1, indeed, should be considered as *coupled spin wave modes* of the two magnetic layers.

3. The spectrum of coupled linear spin wave modes in a magnetic pillar

We consider the case of a two-layer magnetic pillar under the influence of a constant in-plane bias magnetic field that is sufficiently large to guarantee that in the ground state of the system both layers are magnetized in-plane and parallel to the bias field. We linearize equation (1) by the substitution:

$$\mathbf{M}_j(t) = M\left[\mathbf{x} + (\mathbf{m}_j e^{i\omega t} + c.c.)\right] \ . \tag{14}$$

The first term here corresponds to the equilibrium orientation of the magnetization vectors, whereas the second term describes the small-amplitude (linear) spin wave modes. The dimensionless vectors $\mathbf{m}_j$ are orthogonal to the unit vector $\mathbf{x}$ in the linear approximation. Keeping only the terms that are linear in $\mathbf{m}_j$, we get the following eigenvalue problem for the determination of the frequencies ω and profiles $\mathbf{m}_j$ of the coupled spin wave modes of the pillar:

$$i\omega\mathbf{m}_j = \gamma\mathbf{x} \times \left[(H_{ext} - 4\pi M \sum_k \rho_{jk})\mathbf{m}_j + 4\pi M \sum_k \widehat{N}_{jk}\mathbf{m}_k \right] . \quad (15)$$

The above system has non-trivial solutions when

$$\left(\omega^2 - \widetilde{\omega}_1^2\right)\left(\omega^2 - \widetilde{\omega}_2^2\right) = \Phi_{int} \, , \quad (16)$$

where

$$\Phi_{int} = -4\rho_{12}\rho_{21}\omega_M^2 \left(\omega^2 + \rho_{12}\rho_{21}\omega_M^2 - \omega_{h,1}\omega_{h,2} - \frac{\widetilde{\omega}_1^2\widetilde{\omega}_2^2}{4\omega_{h,1}\omega_{h,2}} \right) . \quad (17)$$

Here $\omega_M = 4\pi\gamma M$, $\omega_{h,j} = \gamma h_j$. The field h_j is the *static* magnetic field acting on the j-th layer, i.e. it is the external magnetic field plus the *static* dipolar field created by the *other* layer:

$$h_j = H_{ext} - 4\pi\rho_{jj'} M \; . \quad (18)$$

where $j' = 3 - j$.

The frequency $\widetilde{\omega}_j$ in (16) is the precession frequency in the j-th layer with account of *only the static coupling* to the other layer:

$$\widetilde{\omega}_j = \gamma\sqrt{h_j[h_j + 4\pi(1 - 3\rho_{jj})M]} \; . \quad (19)$$

Clearly, this expression coincides with the Kittel's formula for ferromagnetic resonance frequency with the external field h_j.

The term Φ_{int} in (16) describes the *dynamic* coupling between the two layers. Due to the prefactor $\rho_{12}\rho_{21}$, this term vanishes when the thickness of *either* magnetic layer reduces to zero. Physically, this means that a significant *dynamic* coupling is possible only between the magnetic layers of comparable volumes. To the best of our knowledge, the dynamical coupling described by Φ_{int} was neglected in all the previous theoretical studies of magnetization dynamics in two-layered nano-pillars.

Equation (16) allows one to find an exact analytic solution for the frequencies $\omega_{1,2}$ of the two spatially uniform spin wave modes existing in the considered system. This solution, however, is quite cumbersome. Fortunately, it is possible to derive a relatively simple approximate expression for the spin wave frequencies of the above modes in the form of a traditional Kittel's formula with renormalized values of the saturation magnetization and effective anisotropy fields:

$$\omega_{1,2} = \gamma\sqrt{\left(H_{ext} - \widetilde{H}_{1,2}\right)\left(H_{ext} - \widetilde{H}_{1,2} + 4\pi\widetilde{M}_{1,2}\right)} \, , \quad (20)$$

where the apparent saturation magnetization values are given by

$$\widetilde{M}_1 = \left(1 - 3\frac{\rho_{11}\rho_{21} + \rho_{22}\rho_{12} + 2\rho_{12}\rho_{21}}{\rho_{12} + \rho_{21}} \right) M \, , \quad (21)$$

$$\widetilde{M}_2 = \left(1 - 3\frac{\rho_{11}\rho_{12} + \rho_{22}\rho_{21} - 2\rho_{12}\rho_{21}}{\rho_{12} + \rho_{21}} \right) M \, , \quad (22)$$

and the effective anisotropy fields are equal to

$$\widetilde{H}_1 = 0\,, \tag{23}$$

$$\widetilde{H}_2 = 4\pi(\rho_{12} + \rho_{21})M\ . \tag{24}$$

It should be noted that the frequencies (20) are the *exact* solutions of equation (16) in two limiting cases: (i) when the layers have the same thickness, and (ii) when the thickness of one of the layers tends to zero.

For symmetrical system (both layers have identical thicknesses), the mode with the frequency ω_1 corresponds to symmetric excitations (the magnetization precession in both layers has the same phase), whereas the mode with the frequency ω_2 describes antisymmetric excitations (opposite phases of precession in two interacting layers). When the thicknesses of the layers are different one can still classify the mode ω_1 as a quasi-symmetric mode, and mode ω_2 as a quasi-antisymmetric mode.

The frequencies $\omega_{1,2}$ of the coupled linear excitations in a magnetic pillar are presented in figure 3. One can see that approximate expressions (20) fit the exact solution of the secular equation (16) with high accuracy at small bias magnetic fields. When the bias field reaches the value

$$H_* = \widetilde{H}_2 \frac{4\pi\widetilde{M}_2 - \widetilde{H}_2}{4\pi(\widetilde{M}_1 - \widetilde{M}_2) - 2\widetilde{H}_2}\,, \tag{25}$$

the curves that correspond to the approximate expressions (20) intersect, while the curves that correspond to the exact solution of (16) do not (see figure 3b). The modes described by the secular equation (16) change their symmetry near this point, whereas the modes described by the approximate expressions (20) retain their symmetry. Above the field H_* the approximate solutions (20) again give a reasonably good description of the coupled spin wave modes of a pillar.

One can see from (20) that for a sufficiently small bias field $H_{ext} < \widetilde{H}_2$ ($\widetilde{H}_2 =$ 0.7 kOe for the spectrum presented in figure 3) the frequency of the antisymmetric mode ω_2 becomes imaginary. This means that the ground state corresponding to parallel magnetization of the layers is unstable and the the magnetic pillar will spontaneously relax towards the antiparallel ground state.

As one can see from figure 3a, both spin wave modes of a pillar lie lower than the Kittel's modes of continuous magnetic layers, which corresponds to the apparent decrease of the saturation magnetization. The relative decrease of the saturation magnetization $(\widetilde{M}_{1,2} - M)/M$ with respect to the saturation magnetization of a continuous (unbounded) ferromagnetic film as a function of aspect ratio of the first layer L_1/R is demonstrated in figure 4. One can see that when the difference in the layer thicknesses increases (which means weakening of the dynamic dipolar coupling between the layers) the effective decrease of the saturation magnetization occurs mainly due to the self-demagnetization effects. In the region of the approximately equal thicknesses of the layers, the dynamical coupling leads to a further decrease of the apparent saturation magnetization for the quasi-symmetric mode, but to the increase (compared to the case of the uncoupled layers) of the effective magnetization for the antisymmetric mode.

4. Conclusions

We demonstrated analytically that the magnetodipolar interaction in a two-layered magnetic pillar can be fully described by two diagonal tensors for every layer: one of

Figure 3. (Color online) (a) The frequencies of linear excitations in a two-layered magnetic pillar as a function of the bias magnetic field. Solid lines: exact solutions of the secular equation (16). Triangles and dots: the approximate expressions (20). Dashed and dotted lines: traditional Kittel's expressions for an in-plane magnetized ferromagnetic film [13]: $\omega_{K,1} = \gamma\sqrt{H_{ext}(H_{ext} + 4\pi M)}$ and $\omega_{K,2} = \gamma\sqrt{(H_{ext} - \widetilde{H}_2)(H_{ext} - \widetilde{H}_2 + 4\pi M)}$, respectively. (b) The frequencies $(\omega - \gamma H_{ext})/2\pi$ as functions of the bias magnetic field. Solid lines: exact solutions of the secular equation (16). Triangles and dots: the approximate expressions (20). Parameters used for the calculation: $L_1/R = 1/5$, $L_2/R = 1/10$, $d/R = 1/25$.

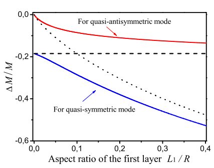

Figure 4. (Color online) Solid lines: relative decrease of the apparent saturation magnetization $(\widetilde{M}_{1,2} - M)/M$ for both quasi-symmetric and quasi-antisymmetric modes with respect to the saturation magnetization of an unbounded ferromagnetic film. Dotted and dashed lines: the effective decrease of the saturation magnetization for the first and the second isolated (uncoupled) layers, respectively. Parameters used for the calculation: $L_2/R = 1/10$, $d/R = 1/25$.

them is the usual tensor of demagnetizing coefficients (self-demagnetization tensor), while the other one is the cross-demagnetization tensor which describes the dipole-dipole interaction between the different layers. Every tensor is fully characterized by only one dimensionless parameter which depends only on the relative geometrical sizes of the pillar's magnetic layers.

The spectrum of coupled oscillations of a two-layered nano-pillar consists of two modes with different frequencies, which can be classified as quasi-symmetric and quasi-antisymmetric modes. For low (high) bias magnetic fields the frequency of the quasi-antisymmetric mode is smaller (larger) than the frequency of the quasi-symmetric mode.

We also demonstrated that the frequencies of coupled spin wave modes of a magnetic pillar can be described by the traditional Kittel's expression with reduced saturation magnetization and renormalized bias field. The magnetodipolar interaction leads to apparent decrease of the saturation magnetization for both modes, but this decrease is more pronounced for the quasi-symmetric mode. For quasi-antisymmetric spin wave mode the apparent reduction of saturation magnetization does not exceed

10 % for realistic nano-pillar parameters, while for the quasi-symmetric mode it can be five times larger.

We believe that the apparent reduction of the static magnetization observed in the spin-torque experiments with magnetic nano-pillars [4, 5, 6, 7, 8] can be, at least in part, attributed to the above described effect of dynamic dipolar interaction between the nano-pillar magnetic layers.

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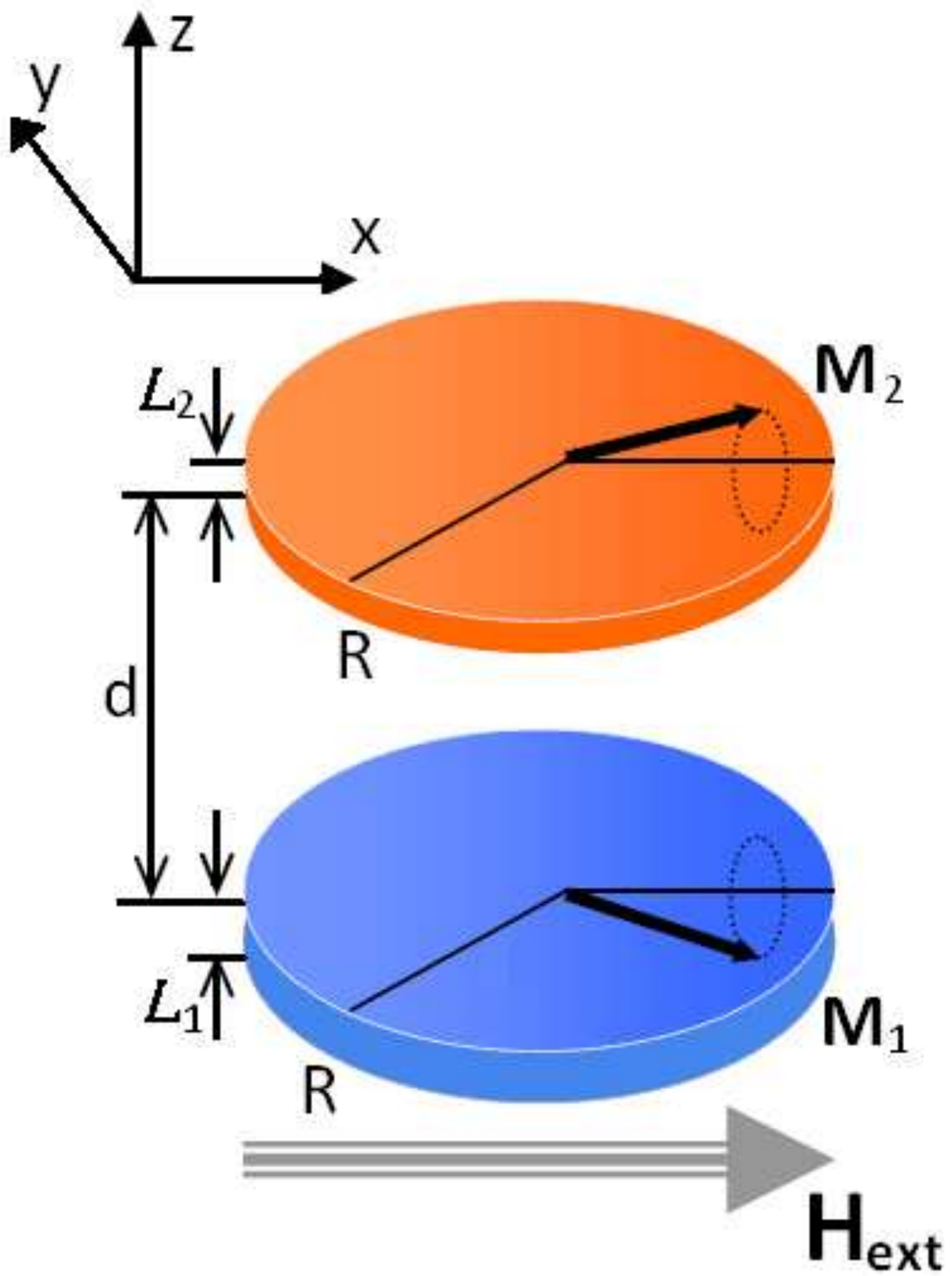
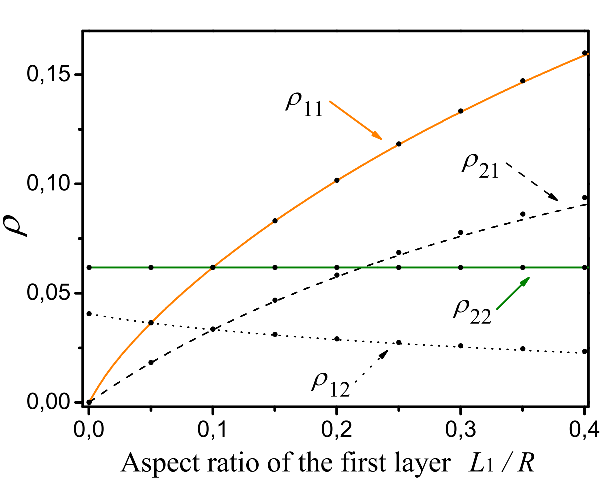
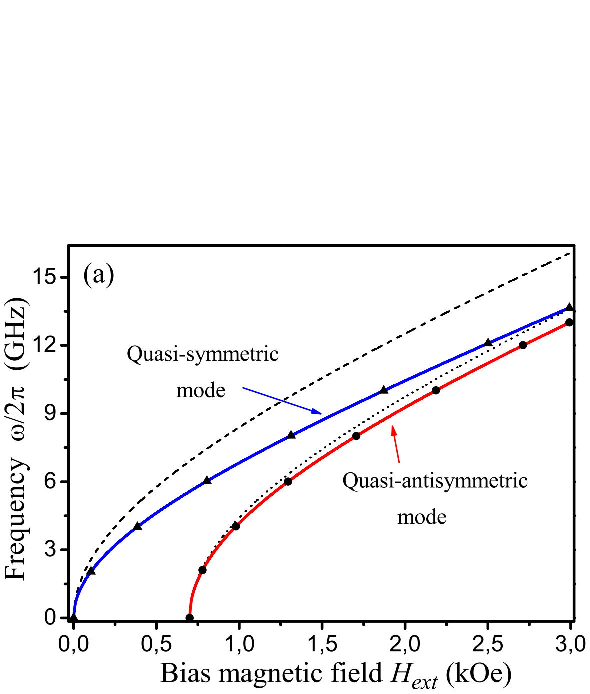
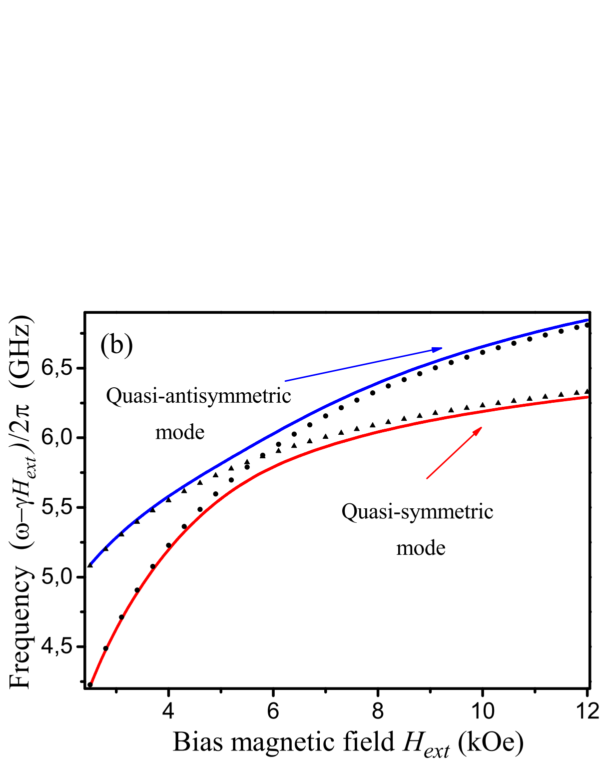

z
y
x
L_2
$\mathbf{M}_2$
R
d
L_1
$\mathbf{M}_1$
R
$\mathbf{H}_{ext}$

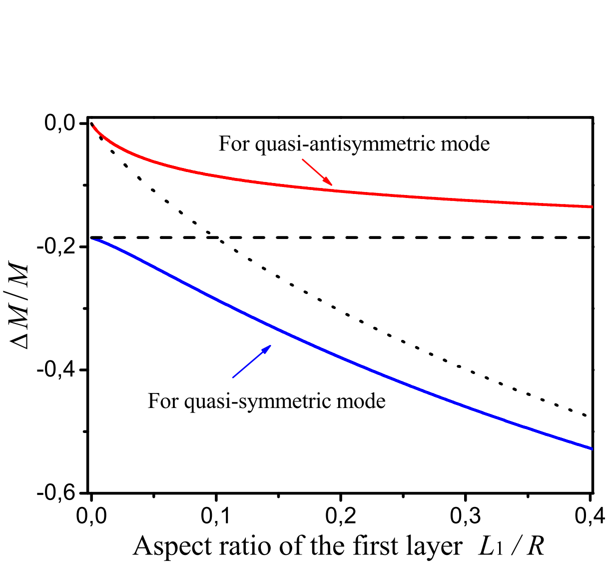


ρ
0,15
0,10
0,05
0,00
0,0
0,1
0,2
0,3
0,4
ρ11
ρ21
ρ22
ρ12
Aspect ratio of the first layer L1 / R


(a)
Quasi-symmetric
mode
Quasi-antisymmetric
mode
Frequency ω/2π (GHz)
0
3
6
9
12
15
Bias magnetic field H_{ext} (kOe)
0,0
0,5
1,0
1,5
2,0
2,5
3,0



(b)
Quasi-antisymmetric
mode
Quasi-symmetric
mode
Frequency $(\omega-\gamma H_{ext})/2\pi$ (GHz)
Bias magnetic field H_{ext} (kOe)
4,5
5,0
5,5
6,0
6,5
4
6
8
10
12



0,0
-0,2
-0,4
-0,6
0,0
0,1
0,2
0,3
0,4
$\Delta M / M$
Aspect ratio of the first layer L_1 / R
For quasi-antisymmetric mode
For quasi-symmetric mode